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evident from several facts. The spectra of air do not in every case show all the lines of oxygen and nitrogen, and occasionally have some not visible in either of them: the spectrum of graphite in oxygen is quite different from those of carbonic oxide. There is even reason to believe that for certain lines the actions of bodies may be antagonistic. The spectrum of mercury electrodes and mercury vapour showed 48 lines, and the author expected that the spectra for any gas with mercury electrodes would add to those of mercury the peculiar lines of that gas, which could thus be certainly determined. In the nitrogen spectrum, however, 20 of the mercurial lines had disappeared, in the hydrogen 18, and in the carbonic oxide 13.

4. The brilliancy or visibility of the lines is very little increased by greatly augmenting the heating power of the discharge. The two halves of the induction machine can be made to act either consecutively for tension, or collaterally for quantity. In the latter case the quantity is doubled, and therefore the heating power quadrupled. When the apparatus is so used, the violet bands are something brighter, but not so much so as to be noticed by an unpractised observer. The red and green show no appreciable difference; but the author is inclined to think the change may be greater in the ultra-violet part. He proposes, however, to repeat the experiment with coils of much greater power as to quantity. If electricity can produce thermic vibrations by its transmission, there seems no *à priori* reason why it cannot produce luminous ones; and no evidence that it cannot is known to him.

It seems to follow from these observations that the tendency to show such lines belongs to matter in general, but that different forms of it have different powers of manifesting that tendency, and that those powers may sometimes interfere. If this be confirmed by further research, the result will be that, though the *electric* spectrum may give useful indications to the analyst, it should never be his sole dependence, or be trusted without full cognizance of the conditions which may affect its indications.

XXV. "On Fermat's Theorem of the Polygonal Numbers."—Second Communication. By the Right Hon. Sir FREDERICK POLLOCK, F.R.S., Lord Chief Baron. Received June 19, 1862.

(Abstract.)

The object of this paper is to show the result of combining the three series (which have been the subject of previous communications) in a square, in such manner that the division into 4 squares of *certain* terms in each series, may produce a division into 4 squares of *every* term of other series, and thus each term in the whole square will at last be divided into 4 squares, and the first term will be so divided into 4 square numbers that two of the roots will be equal to each other; two of them will differ by 1, and the algebraic sum of all the roots will be equal to 1.

It is not offered (at present) as a *proof* that it *must* be so, but as a *method* by which that result may always (in fact) be obtained.

If any odd number $2n+1$ be increased by 2, 4, 6, 8, 10, &c., the $(2n+1)$th term will be $(2n+1)^2$; other terms will have a distinct arithmetic relation to n, and $n+1$, and the whole series will be such that, if the pth term can be divided into square numbers whose roots shall equal $2p-1$, then every term of the whole series can be so divided that the roots of the $(p+1)$th term will be $2p+1$, and so on through the whole series.

Let $\overset{1}{27}, \overset{3}{29}, \overset{5}{33}, \overset{7}{39}, \overset{9}{47}, \overset{11}{57}$, &c. be such a series, with the odd numbers as indices of the sums of the roots, $39={}^{2}\overline{-2\overset{3}{+}1\overset{2}{+}3\overset{2}{+}5}$, and the sum of the roots is 7, and the differences of the roots, placed in arithmetic order, will be 3 . 2 . 2; then 29 will have roots with the same differences, the sum being 3, ${}^{2}\overline{-3\overset{3}{,}\ 0\overset{2}{,}\ 2\overset{2}{,}\ 4}=29$, and 57 will have roots ${}^{2}\overline{-1\overset{3}{,}\ 2\overset{2}{,}\ 4\overset{2}{,}\ 6}=57$. The other numbers in the series will have the differences reversed, but the sums of the roots will be respectively as the odd numbers placed as the index of each.

If any odd number be increased by 4, 8, 12, 16, &c., so as to form a series, $2n+1$, $2n+5$, $2n+13$, $2n+25$, pth term $(2n+(p-)^2, +p^2)$, it will have in the first term 4 roots, 2 of which differ by 1; in the 2nd term, 4 roots, 2 of which differ by 3; in the 3rd term 4 roots, 2 of which differ by 5; in the nth term 4 roots, 2 of which differ by $2n-1$; the other two roots will be common to all the terms. If these odd numbers, 1, 3, 5, &c., be made indices of the 1st, 2nd, 3rd, &c. terms, and any one term can be found having 2 roots differing by the index of that term, then the roots of all the other terms may be found. Let 27 increase by 4, 8, 12, &c.,

$$\overset{1}{27}\quad \overset{3}{31}\quad \overset{5}{39}\quad \overset{7}{51}\quad \overset{9}{67}, \text{\&c.};$$

but $39 = {}^{2}\overline{2,\ 3,\ 1,\ 5}$, and -2, 3 differ by 5. Then the terms of the whole series may be divided into 4 squares, 2 of which will be common to all the terms, and the other 2 will have the difference pointed out by the index. The roots are placed below each term, and the middle roots are common to all the terms :

$$\begin{array}{cccccc}
1 & 3 & 5 & 7 & 9 & 11 \\
27 & 31 & 39 & 51 & 67 & 87, \text{ \&c.} \\
0,1,5,1 & -1,1,5,2 & -2,1,5,3 & -3,1,5,4 & -4,1,5,5 & -5,1,5,6
\end{array}$$

If an odd number $(2n+1)$ be increased by 2, 6, 10, 14, &c., and $2n+1, 2n+3, 2n+9, 2n+19, 2n+(p-1)^2, (p-1)^2, 1$ (pth term) be the resulting series, then, if the even numbers (beginning with 0) be made indices, and any term in the series can be divided into 4 squares, 2 of them having their roots with the algebraic difference pointed out by the index, then the other 2 roots will be common to all the terms, and in a similar manner all the terms will have roots corresponding with the index of each term.

The series $2n+1$, $2n+5$, $2n+13$, $2n+25$, &c. will have for its nth term $n^2, +n^2+\textcircled{1}$, for its nth term is obviously $2n+(n-1)^2 + n^2 = n^2+n^2+\textcircled{1}$; the $(n-1)$th term will be $(n-1)^2, +(n-1)^2+\textcircled{3}$; and going backwards to the first term, the roots $(n-1)$, $(n-1)$ decrease by 1, and the arithmetic number increases by 2; but this obtains beyond the first term into a continuation of the series backwards; thus,

$$\begin{array}{cccccccc}
23 & 15 & 11 & 11 & 15 & 23 & 35 & 51 \\
2\textcircled{15}2, & 1\textcircled{13}1 & 0\textcircled{11}0 & 1\textcircled{9}1 & 2\textcircled{7}2 & 3\textcircled{5}3 & 4\textcircled{3}4 & 5\textcircled{1}5.
\end{array}$$

Instead of this mode of continuing the roots and arithmetic numbers, they may be applied thus :—

$$\begin{array}{cccccccc}
1 & 3 & 5 & 7 & 9 & 11 & 13 & 17 \\
{}_0\textcircled{11}_0 & {}_1\textcircled{13}_1 & {}_2\textcircled{15}_2 & {}_3\textcircled{17}_3 & {}_4\textcircled{19}_4 & {}_5\textcircled{21}_5 & {}_6\textcircled{23}_6 & {}_7\textcircled{25}_7 \\
11 & 15 & 23 & 35 & 51 & 71 & 95 & 123 \\
{}^1\textcircled{9}^1 & {}^2\textcircled{7}^2 & {}^3\textcircled{5}^3 & {}^4\textcircled{3}^4 & {}^5\textcircled{1}^5 & & &
\end{array}$$

And whenever the arithmetic number is of the form $2a^2+2a+2b^2+1$, (that is) is the sum of 2 triangular numbers multiplied by 2 and increased by 1, then, by altering the even squares, the term may be made to consist of 4 squares, as to which the roots of 2 of them will differ by 1.

As an example, $51 = 4^2 + 4^2 + 19$
$= 3^2 + 5^2 + (19 - 2)\ 17$
$= 2^2 + 6^2 + (19 - 8)\ 11$
$= 1^2 + 7^2 + (19 - 18)\ 1^2.\ 0^2.$

The series $2n+1$, $2n+3$, $2n+9$, $2n+19$, &c., has similar properties, not necessary to be stated in detail in order to understand the effect of placing the series in a square.

	1	3	5	7	9	11
0	1 11	3 15	5 23	7 35 −1, 0, 3, 5	9 51 0, 1, 5, 5	11 71
2	1 3 13	1 5 17	3 7 25 −2, −1, 2, 4	5 9 37 1, 2, 4, 4	7 11 53 0, 1, 4, 6	9 13 73
4	3 5 19 0, 1, 3, 3	1 7 23 −3, −2, 1, 3	1 9 31 2, 3, 3, 3	3 11 43	5 13 59 0, 1, 3, 7	7 15 79
6	5 7 29 −4, −3, 0, 2	3 9 33 3, 4, 2, 2	1 11 41	1 13 53	3 15 69 0, 1, 2, 8	5 17 89
8	7 9 43 4, 5, 1, 1	5 11 47	3 13 55	1 15 67	1 17 83 0, 1, 1, 9	3 19 103
10	9 11 61 5, 6, 0, 0	7 13 65	5 15 73	3 17 85	1 19 101 0, 1, 0, 10	1 21 121

If an odd number, as 11 (see diagram, No. 1), be placed in the upper line on the left-hand square of a larger square divided into small squares, and be made the first term of a series, increasing by 4, 8, 12, &c., and then each term of that series be made the first term of a series increasing by 2, 6, 10, 14, &c., the square will be completed, and every odd number in the small squares will be a term in both series; but if a diagonal be drawn from 11 to 121, and parallel from

13 to 101, and beginning with 13, a number be taken from each line alternately, a series of the first kind will be discovered, viz. one increasing by 2, 4, 6, 8, 10, &c., and every term in that series will become the first term of another similar series; so that every odd number in the small squares (except those in the top row) will be terms in 2 such series, and the indices will be as marked in the right-hand upper corner of each square; the numbers in the margin 1, 3, 5, 7, &c., and 0, 2, 4, 5, &c., are the indices of all the series, parallel to them.

Now $51 = {}^{2}\overline{0, 1, 5, 5.}$ No two roots differ by 9; but $5=5$; and $5+5-1=9$. As $5=5$, all the numbers below 51, that is 53, 59, &c., are divided into 4 square numbers, whose roots appear in the diagram. Again, $35=4^2$, $4^2+\textcircled{3}=3^2$, 5^2+1, and $35={}^{2}\overline{-1, 0, 3, 5,}$ which gives 7, the index of 35, as a term in the series increasing by 2, 4, 6, &c.; and therefore every term may be resolved into 4 squares; but as one series crosses a set of series, it at length furnishes the index, thus, $\underset{-0,\,-1,\,5,\,5}{51,}$ $\underset{-1,\,-2,\,4,\,4}{37,}$ $\underset{-2,\,-3,\,3,\,3}{31,}$ on arriving at 31. $2+3=5$, the index of 31 as a term in the series increasing by 4, 8, 12, &c.; and 19 therefore equals 0, 1, 3, 3; and -1, $3=4$ (the index of 19 as a term in the 3rd series); $11=0, 3, 1, 1$.

The method by which the division of certain numbers into 4 squares is here accomplished applies to all numbers of the form $2n+1$; but as the first term increases, the methods also multiply, so as to afford increasing means of division, which must be the subject of a future communication.

XXVI. "On the Oxidation and Disoxidation effected by the Alkaline Peroxides." By B. C. Brodie, Esq., F.R.S., Professor of Chemistry in the University of Oxford. Received June 19, 1862.

(Abstract.)

A preliminary notice containing an abstract of the greater portion of this paper has already appeared*.

Having shown that the alkaline peroxides are capable of acting either as agents of oxidation or reduction, the author proceeds to

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* See 'Proceedings,' vol. xi. p. 442.